{202} (ΑΣΚΗΣΗ 2η)

α. Μέσω της αντικατάστασης $u = \pi - x$

ΝΔΟ

$$\int_0^{\pi} x \cdot f(\eta\mu x)dx = \frac{\pi}{2}\int_0^{\pi} f(\eta\mu x)\,dx$$

β. Να υπολογιστεί το ολοκλήρωμα:

$$\int_0^{\pi} \frac{x \cdot \eta\mu x}{3+\eta\mu^2 x}\,dx$$

ΛΥΣΗ

α. $u = \pi - x \Rightarrow dx = -du$ ενώ $x = \pi - u$

- Για $x = 0 \rightsquigarrow u = \pi$
- Για $x = \pi \rightsquigarrow u = 0$

$$\int_0^{\pi} x\, f(\eta\mu x)\,dx = \int_{\pi}^{0} (\pi-u)\cdot f(\eta\mu(\pi-u))\cdot(-du) =$$

$$= \int_0^{\pi} (\pi-u)\cdot f(\eta\mu u)\cdot du = \int_0^{\pi}\left(\pi f(\eta\mu u) - u f(\eta\mu u)\right)du =$$

$$= \int_0^{\pi} \pi \cdot f(\eta\mu u)\,du - \int_0^{\pi} u f(\eta\mu u)\,du =$$

$$= \pi \cdot \int_0^{\pi} f(\eta\mu u)\,du - \int_0^{\pi} u\, f(\eta\mu u)\,du =$$

$$= \pi \int_0^{\pi} f(\eta\mu x)\,dx - \int_0^{\pi} x \cdot f(\eta\mu x)dx \Rightarrow$$

$$\Rightarrow 2\int_0^{\pi} x\, f(\eta\mu x) = \pi \int_0^{\pi} f(\eta\mu x)\,dx \Rightarrow$$

$$\Rightarrow \int_0^{\pi} x\, f(\eta\mu x) = \frac{\pi}{2}\int_0^{\pi} f(\eta\mu x)\,dx$$

β. $\displaystyle\int_0^{\pi} \frac{x\cdot \eta\mu x}{3+\eta\mu^2 x}\,dx = \int_0^{\pi} x\left(\frac{\eta\mu x}{3+\eta\mu^2 x}\right) dx \overset{(\alpha.)}{=\!=}$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{\eta\mu x}{3+\eta\mu^2 x}\,dx = \frac{\pi}{2}\int_0^{\pi} \frac{\eta\mu x}{3+1-\sigma\upsilon\nu^2 x}\,dx =$$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{\eta\mu x}{4-\sigma\upsilon\nu^2 x}\,dx$$

Θέτω $\sigma\upsilon\nu x = k \Rightarrow dx = -\dfrac{dk}{\eta\mu x}$

- $x=\pi$, $k=-1$
- $x=0$, $k=1$

Άρα, $\displaystyle\frac{\pi}{2}\int_0^{\pi} \frac{\eta\mu x}{4-\sigma\upsilon\nu^2 x}\,dx = \frac{\pi}{2}\int_1^{-1} \frac{\cancel{\eta\mu x}}{4-k^2}\left(-\frac{dk}{\cancel{\eta\mu x}}\right) =$

$$= \frac{\pi}{2}\int_{-1}^{1} \frac{1}{4-k^2}\,dk.$$

$$\frac{1}{4-k^2} = \frac{1}{(2-k)(2+k)} = \frac{A}{(2-k)} + \frac{B}{(2+k)} \Rightarrow$$

$$\Rightarrow 1 = (A-B)k + 2A + 2B$$

$A-B=0$ και $2A+2B=1$

$A=\frac{1}{4}$ και $B=\frac{1}{4}$

$$\frac{\pi}{2}\int_{-1}^{1} \frac{1}{4-k^2}\,dk = \frac{\pi}{2}\int_{-1}^{1} \frac{1/4}{(2-k)}\,dk + \int_{-1}^{1} \frac{1/4}{(2+k)}\,dk =$$

$$= \frac{\pi}{2}\left(\frac{1}{4}\left(-\log(2-k)\right)\Big|_{-1}^{1} + \frac{1}{4}\log(2+k)\Big|_{-1}^{1}\right) =$$

$$= \frac{\pi}{8}\cdot\left(-\log 1 + \log 3 + \log 3 + \log 1\right) = \frac{\pi}{4}\cdot\log 3.$$